# CHAPTER 2

# THICK CYLINDERS

## 2.1. INTRODUCTION

If the ratio of thickness to internal diameter of a cylindrical shell is less than about 1/20, the cylindrical shell is known as thin cylinders. For them it may be assumed with reasonable accuracy that the hoop and longitudinal stresses are constant over the thickness and the radial stress is small and can be neglected. If the ratio of thickness to internal diameter is more than 1/20, then cylindrical shell is known as thick cylinders.

## 2.2. Difference in treatment between thin and thick Cylinders- basic assumptions

The theoretical treatment of thin cylinders assumes that the hoop stress is constant across the thickness of the cylinder wall (Fig. 1), and also that there is no pressure gradient across the wall. Neither of these assumptions can be used for thick cylinders for which the variation of hoop and radial stresses is shown in Fig. 2, their values being given by the Lamé equations:

$$\sigma_H = A + \frac{B}{r^2} \quad \textbf{and} \quad \sigma_r = A - \frac{B}{r^2}$$

The hoop stress in case of a thick cylinder will not be uniform across the thickness.
Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.
Development of the theory for thick cylinders is concerned with sections remote from the ends since distribution of the stresses around the joints makes analysis at the ends particularly complex. For central sections the applied pressure system which is normally applied to thick cylinders is symmetrical, and all points on an annular element of the cylinder wall will be displaced by the same amount, this amount depending on the radius of the element. Consequently there can be no shearing stress set upon transverse planes and stresses on such planes are therefore principal stresses. Similarly, since the radial shape of the cylinder is maintained there are no shears on radial or tangential planes, and again stresses on such planes are principal stresses. Thus, consideration of any element in the wall of a thick cylinder involves, in general, consideration of a mutually perpendicular, tri-axial, principal

stress system, the three stresses being termed **radial**, **hoop (tangential or circumferential) and longitudinal (axial) stresses.**

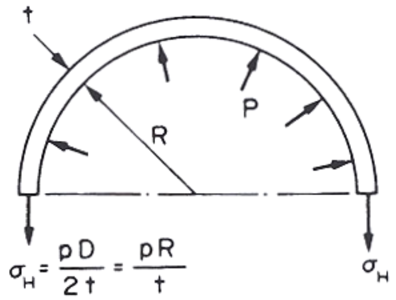


Fig.1. Thin cylinder subjected to internal pressure.

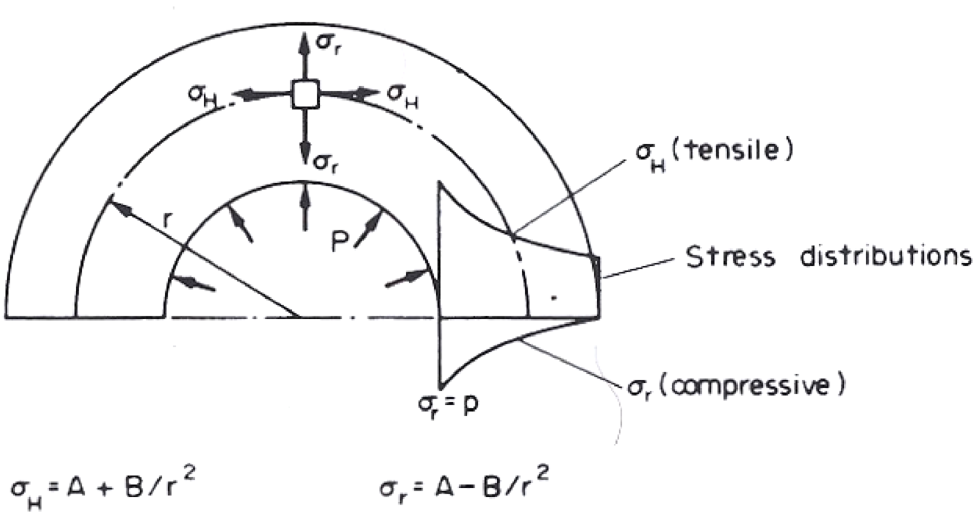


Fig.2. Thick cylinder subjected to internal pressure.

**2.3. <u>Thick cylinder-internal pressure only</u>**

Consider now the thick cylinder shown in Fig. 3 subjected to an internal pressure P, the external pressure being zero.

The two known conditions of stress which enable the Lamé constants A and B to be determined are:

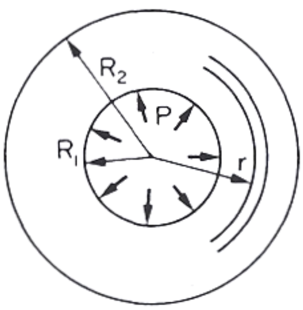


Fig.3. Cylinder cross-section.

Atr=$R_1$ $\sigma_r$= - P and atr=$R_2$ $\sigma_r = 0$

NB. - The internal pressure is considered as a negative radial stress since it will produce aradial compression (i.e. thinning) of the cylinder walls and the normal stress convention takes compression as negative.

Substituting the above conditions in equation of radial stress:

$$-P = A - \frac{B}{R_1^2}$$

$$0 = A - \frac{B}{R_2^2}$$

i.e. $$A = \frac{PR_1^2}{(R_2^2 - R_1^2)} \quad \text{and} \quad B = \frac{PR_1^2 R_2^2}{(R_2^2 - R_1^2)}$$

$\therefore$ $$\text{radial stress } \sigma_r = A - \frac{B}{r^2}$$

$$= \frac{PR_1^2}{(R_2^2 - R_1^2)}\left[1 - \frac{R_2^2}{r^2}\right]$$

$$= \frac{PR_1^2}{(R_2^2 - R_1^2)}\left[\frac{r^2 - R_2^2}{r^2}\right] = -P\left[\frac{(R_2/r)^2 - 1}{k^2 - 1}\right]$$

where $k$ is the diameter ratio $D_2/D_1 = R_2/R_1$

and $$\text{hoop stress } \sigma_H = \frac{PR_1^2}{(R_2^2 - R_1^2)}\left[1 + \frac{R_2^2}{r^2}\right]$$

$$= \frac{PR_1^2}{(R_2^2 - R_1^2)}\left[\frac{r^2 + R_2^2}{r^2}\right] = P\left[\frac{(R_2/r)^2 + 1}{k^2 - 1}\right]$$

These equations yield the stress distributions indicated in Fig. 2 with maximum values of both $\sigma_r$, and $\sigma_H$ at the inside radius.

### 2.4. Longitudinal stress

Consider now the cross-section of a thick cylinder with closed ends subjected to an internal pressure $P_1$ and an external pressure $P_2$ (Fig.4).

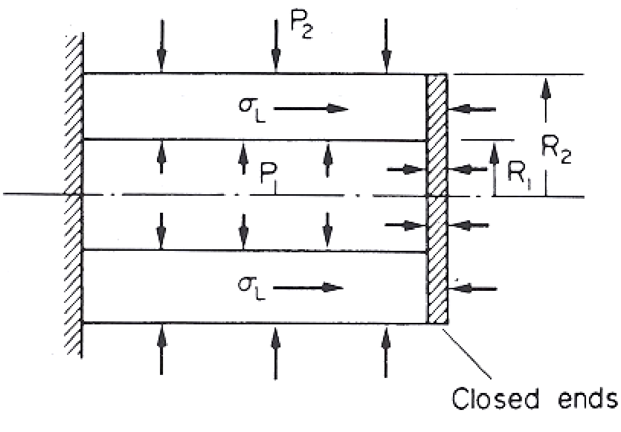


Fig. 4.Cylinder longitudinal section.

For horizontal equilibrium:

$$P_1 \times \pi R_1^2 - P_2 \times \pi R_2^2 = \sigma_L \times \pi(R_2^2 - R_1^2)$$

where$\sigma_L$ is the longitudinal stress set up in the cylinder walls,

$$\therefore \qquad \text{longitudinal stress } \sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{R_2^2 - R_1^2}$$

i.e. a constant.

For combined internal and external pressures, the relationship $\sigma_L = A$ also applies.

## 2.5. Maximum shear stress

It has been stated in §1 that the stresses on an element at any point in the cylinder wall are principal stresses.

It follows, therefore, that the maximum shear stress at any point will be given by eqn. as

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

i.e. half the difference between the greatest and least principal stresses. Therefore, in the case of the thick cylinder, normally,

$$\tau_{max} = \frac{\sigma_H - \sigma_r}{2}$$

Since $\sigma_H$ is normally tensile, whilst $\sigma_r$ is compressive and both exceed $\sigma_L$in magnitude.

$$\therefore \qquad \tau_{max} = \frac{1}{2}\left[\left(A + \frac{B}{r^2}\right) - \left(A - \frac{B}{r^2}\right)\right]$$

$$\tau_{max} = \frac{B}{r^2}$$

The greatest value of $\tau_{max}$ thus normally occurs at the inside radius where $r = R_1$.

## 2.6. <u>Change of cylinder dimensions</u>

### (a) Change of diameter

The diametral strain on a cylinder is equal to the hoop orcircumferential strain.

Therefore, change of diameter = diametral strain x original diameter

= circumferential strain x original diameter

With the principal stress system of hoop, radial and longitudinal stresses, all assumed tensile,the circumferential strain is given by

$$\varepsilon_H = \frac{1}{E}[\sigma_H - \nu\sigma_r - \nu\sigma_L]$$

Thus the change of diameter at any radius r of the cylinder is given by

$$\Delta D = \frac{2r}{E}[\sigma_H - \nu\sigma_r - \nu\sigma_L]$$

### (b) Change of length

Similarly, the change of length of the cylinder is given by

$$\Delta L = \frac{L}{E}[\sigma_L - \sigma\nu_r - \nu\sigma_H]$$

## 2.7. <u>Compound cylinders</u>

To obtain a more uniform hoop stress distribution, cylinders are often built up by shrinking one tube on to the outside of another. When the outer tube contracts on cooling the inner tube is brought into a state ofcompression. The outer tube will conversely be brought into a state of tension. If this compound cylinder is now subjected to internal pressure the resultant hoop stresses will be the algebraic sum of those resulting from internal pressure and those resulting from shrinkage as drawn in Fig.5 ; thus a much smaller total fluctuation of hoop stress is obtained. A similar effect is obtained if a cylinder is wound with wire or steel tape under tension.

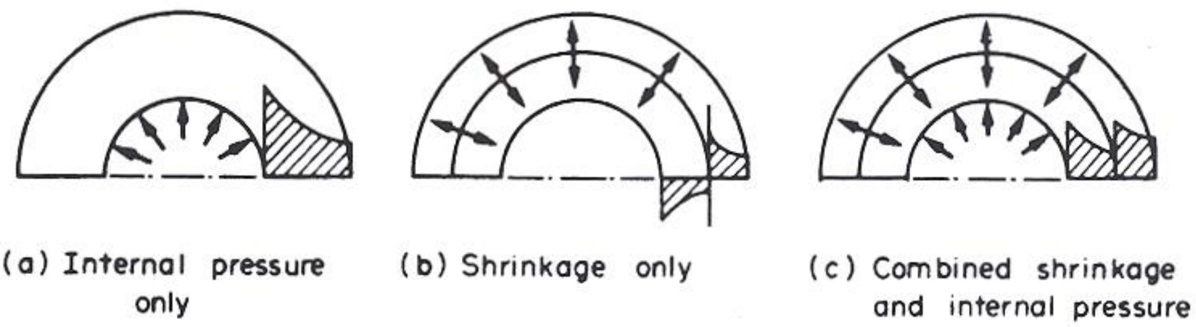


Fig. 5.Compound cylinders - combined internal pressure and shrinkage effects.

**(a) Same materials**

The method of solution for compound cylinders constructed from similar materials is to break the problem down into three separate effects:

(a) shrinkage pressure only on the inside cylinder;

(b) shrinkage pressure only on the outside cylinder;

(c) internal pressure only on the complete cylinder (Fig.6).

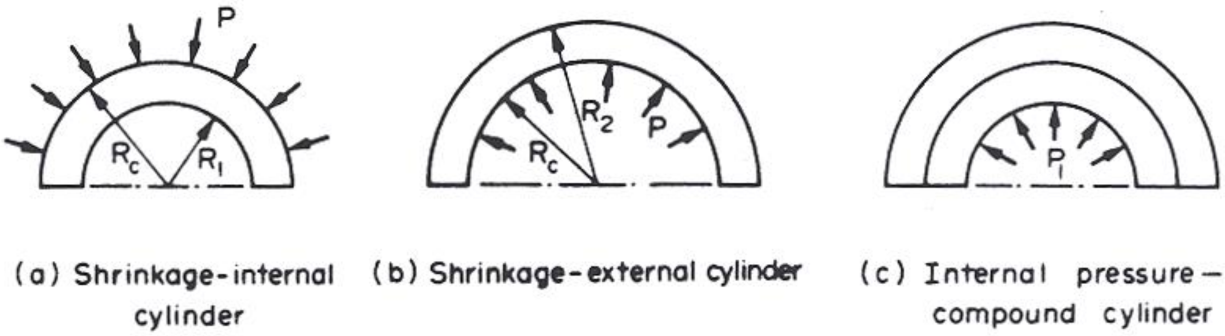


Fig. 6.Method of solution for compound cylinders.

For each of the resulting load conditions there are two known values of radial stress which enable the Lamé constants to be determined in each case:

i.e. condition (a) shrinkage - internal cylinder:

At $r = R_1$, $\sigma_r = 0$

At $r = R_c \sigma_r = - p$ (compressive since it tends to reduce the wall thickness)

condition (b) shrinkage - external cylinder:

At $r = R_2$, $\sigma_r = 0$

At $r = R_c$, $\sigma_r = -p$

condition (c) internal pressure - compound cylinder:

At $r = R_2$, $\sigma_r = 0$

At $r = R_1$, $\sigma_r = -P_1$

Thus for each condition the hoop and radial stresses at any radius can be evaluated and the principle of superposition applied, i.e. the various stresses are then combined algebraically to produce the stresses in the compound cylinder subjected to both shrinkage and internal pressure. In practice this means that the compound cylinder is able to withstand greater internal pressures before failure occurs or, alternatively, that a thinner compound cylinder (with the associated reduction in material cost) may be used to withstand the same internal pressure as the single thick cylinder it replaces.

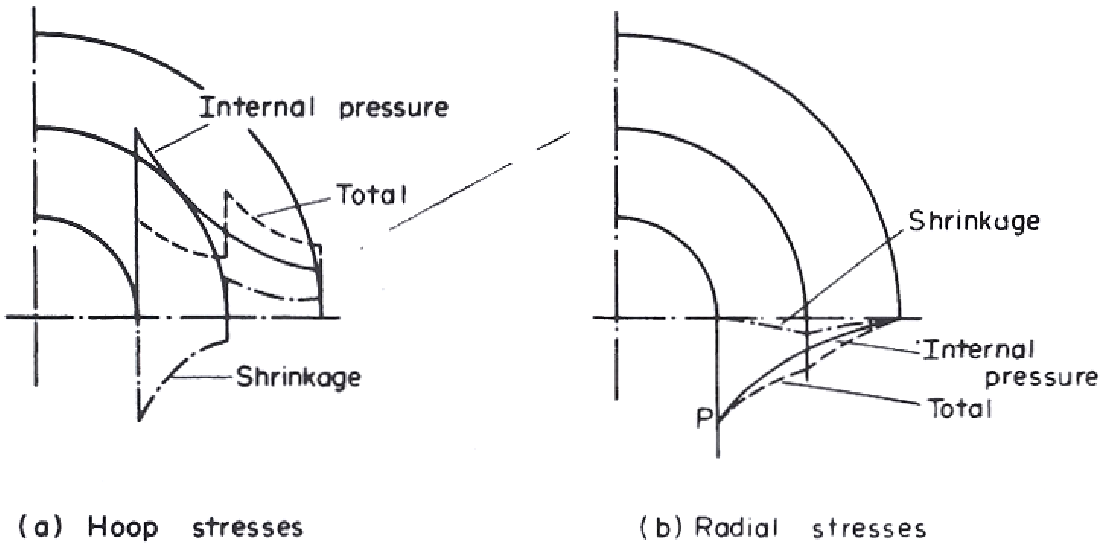


Fig.7. Distribution of hoop and radial stresses through the walls of a compound cylinder.

**(b) Different materials**

The value of the shrinkage or interference allowance for compound cylinders constructed from cylinders of different materials is given by eqn.

**Total interference or shrinkage allowance**
$$= \left[ \frac{1}{E_1}(\sigma_{H_o} + \nu_1 p) - \frac{1}{E_2}(\sigma_{H_i} + \nu_2 p) \right] r$$

The value of the shrinkage pressure set up owing to a known amount of interference can then be calculated as with the standard compound cylinder treatment, each component cylinder being considered separately subject to the shrinkage pressure.

Having constructed the compound cylinder, however, the treatment is different for the analysis of stresses owing to applied internal and/or external pressures. Previously the compound cylinder has been treated as a single thick cylinder and, e.g., a single Lamé line drawn across both cylinder walls for solution. In the case of cylinders of different materials, however, each component cylinder must be considered separately as with the shrinkage effects. Thus, for a known internal pressure $P_1$ which sets up a common junction pressure p, the Lamé line solution takes the form shown in Fig. 8.

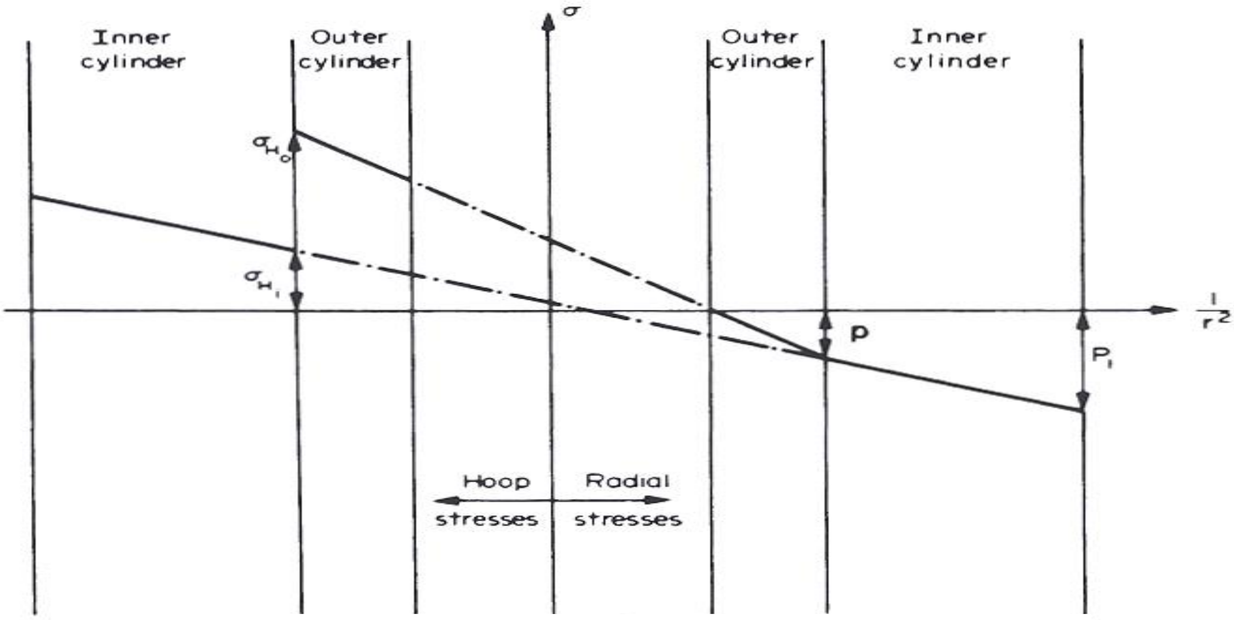


Fig.8. Graphical solution for compound tubes of different materials.

For a full solution of problems of this type it is often necessary to make use of the equality of diametral strains at the common junction surface, i.e. to realize that for the cylinders to maintain contact with each other the diametral strains must be equal at the common surface.

Now **diametral strain = circumferential strain**

$$= \frac{1}{E}[\sigma_H - \nu\sigma_r - \nu\sigma_L]$$

Therefore at the common surface, ignoring longitudinal strains and stresses,

$$\frac{1}{E}[\sigma_{H_o} - \nu_o\sigma_r] = \frac{1}{E_i}[\sigma_{H_i} - \nu_i\sigma_r]$$

Where $E_o$ and $v_o$= Young's modulus and Poisson's ratio of outer cylinder,

$E_i$ and $v_i$ = Young's modulus and Poisson's ratio of inner cylinder,

$\sigma_r$= - p = radial stress at common surface,

and$\sigma_{Ho}$= (as before) the hoop stresses at the common surface for the outer and inner cylinders respectively.

## EXAMPLES

**1.** A thick cylinder of 100 mm internal radius and 150 mm external radius is subjected to an internal pressure of 60 MN/m$^2$ and an external pressure of 30 MN/m$^2$. Determine the hoop and radial stresses at the inside and outside of the cylinder together with the longitudinal stress if the cylinder is assumed to have closed ends.

### Solution

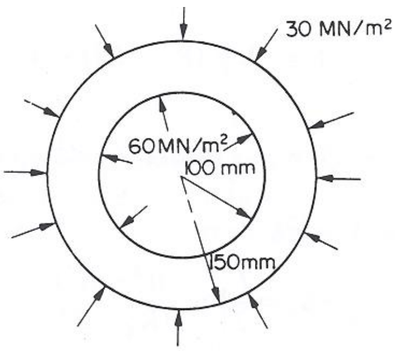

Fig. 9

**(a): analytical**

The internal and external pressures both have the effect of decreasing the thickness of the cylinder; the radial stresses at both the inside and outside radii are thus compressive, i.e. negative (Fig.9).

$\therefore$ at $r = 0.1$ m, $\sigma_r = -60$ MN/m$^2$

and at $r = 0.15$ m, $\sigma_r = -30$ MN/m$^2$

Therefore, from eqn. of radial stress with stress units of MN/m$^2$,

$$-60 = A - 100B \quad \ldots(1)$$

and

$$-30 = A - 44.5B \quad \ldots(2)$$

Subtracting (2) from (1), $-30 = -55.5B$

$$B = 0.54$$

Therefore, from (1), $A = -60 + 100 \times 0.54$

$$A = -6$$

Therefore, at r = 0.1 m, from eqn. hoop stress,

$$\sigma_H = A + \frac{B}{r^2} = -6 + 0.54 \times 100$$

$$= \mathbf{48\ MN/m^2}$$

and at $r = 0.15$ m, $\sigma_H = -6 + 0.54 \times 44.5 = -6 + 24$

$$= \mathbf{18\ MN/m^2}$$

The longitudinal stress is given by

$$\sigma_L = \frac{P_1R_1^2 - P_2R_2^2}{(R_2^2 - R_1^2)} = \frac{(60 \times 0.1^2 - 30 \times 0.15^2)}{(0.15^2 - 0.1^2)}$$

$$= \frac{10^2(60 - 30 \times 2.25)}{1.25 \times 10^2} = -\mathbf{6\ MN/m^2} \quad \text{i.e. compressive}$$

**Solution (b)** graphical

The graphical solution is shown in Fig. 10, where the boundaries of the cylinder are given by

$$\frac{1}{r^2} = 100 \textit{ for the inner radius where } r = 0.1\text{ m}$$

$$\frac{1}{r^2} = 44.5 \text{ for the outer radius where } r = 0.15\text{ m}$$

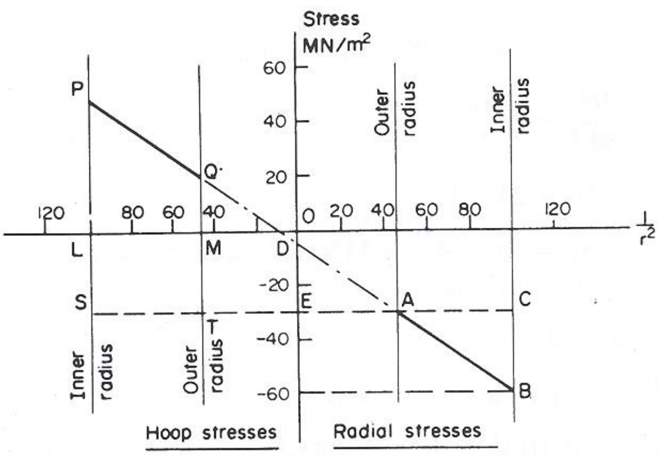


Fig. 10

The two conditions which enable the Lamé line to be drawn are the same as those used above for the analytical solution,

i.e. $$\sigma_r = -60\ \mathrm{MN/m^2} \quad \text{at} \quad r = 0.1\ \mathrm{m}$$
$$\sigma_r = -30\ \mathrm{MN/m^2} \quad \text{at} \quad r = 0.15\ \mathrm{m}$$

The hoop stresses at these radii are then given by points P and Q on the graph. For complete accuracy these values should be calculated by proportions of the graph thus:-

$$\frac{PS}{100+44.5} = \frac{CB}{100-44.5}$$

$\therefore$ $$\frac{PL+LS}{144.5} = \frac{30}{55.5}$$

by similar triangles PAS and BAC

i.e. hoop stress at radius r = 0.1 m

$$= PL = \frac{30 \times 144.5}{55.5} - LS$$
$$= 78 - 30 = \mathbf{48\ MN/m^2}$$

Similarly the hoop stress at radius r = 0.15 m is QM and given by the similar triangles QAT and BAC,

i.e. $$\frac{QM+MT}{44.5+44.5} = \frac{30}{55.5}$$
$$QM = \frac{30 \times 89}{55.5} - 30$$
$$= 48 - 30 = \mathbf{18\ MN/m^2}$$

The longitudinal stress $\sigma_L$ = the intercept on the σ axis (which is negative)

$$= DO = OE - DE = 30 - DE$$

Now $$\frac{DE}{44.5} = \frac{30}{55.5}$$

$\therefore$ $$DE = 24$$

$\therefore$ $$\sigma_L = 30 - 24 = \mathbf{6\ MN/m^2} \quad \text{compressive}$$

**2.** An external pressure of 10 $MN/m^2$ is applied to a thick cylinder of internal diameter 160 mm and external diameter 320 mm. If the maximum hoop stress permitted on the inside wall of the cylinder is limited to 30 $MN/m^2$, what maximum internal pressure can be applied assuming the cylinder has closed ends? What will be the change in outside diameter when this pressure is applied? E = 207 $GN/m^2$, $\nu = 0.29$.

**Solution (a):** analytical

The conditions for the cylinder are:

When r = 0.08 m, $\sigma_r = -p$ and $\frac{1}{r^2} = 156$

When r = 0.16 m, $\sigma_r = -10$ $MN/m^2$ and $\frac{1}{r^2} = 39$ when

and when r = 0.08 m $\sigma_H = 30$ $MN/m^2$

Since the maximum hoop stress occurs at the inside surface of the cylinder.

Using the latter two conditions in eqns. with units of $MN/m^2$,

$$-10 = A - 39B \quad \text{...(1)}$$

$$30 = A + 156B \quad \text{...(2)}$$

Subtracting (1) from (2),

$$40 = 195B \qquad \therefore \; B = 0.205$$

Substituting in (1),

$$A = -10 + (39 \times 0.205)$$
$$= -10 + 8 \qquad \therefore \; A = -2$$

Therefore, at r = 0.08, from eqn. below,

$$\sigma_r = -p = A - 156B$$
$$= -2 - 156 \times 0.205$$
$$= -2 - 32 = \mathbf{-34\ MN/m^2}$$

i.e. the allowable internal pressure is 34 $MN/m^2$.

We know that the change in diameter is given by

$$\Delta D = \frac{2r_0}{E}\,(\sigma_H - \nu\sigma_r - \nu\sigma_L)$$

Now at the outside surface

$$\sigma_r = -10\ \text{MN/m}^2 \quad \text{and} \quad \sigma_H = A + \frac{B}{r^2}$$

$$= -2 + (39 \times 0.205)$$

$$= -2 + 8 = 6\ \text{MN/m}^2$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)} = \frac{(34 \times 0.08^2 - 10 \times 0.16^2)}{(0.16^2 - 0.08^2)}$$

$$= \frac{(34 \times 0.64 - 10 \times 2.56)}{(2.56 - 0.64)} = \frac{21.8 - 25.6}{1.92}$$

$$= -\frac{3.8}{1.92} = 1.98\ \text{MN/m}^2\ \text{compressive}$$

$$\therefore \qquad \Delta D = \frac{0.32}{207 \times 10^9}\,[6 - 0.29(-10) - 0.29(-1.98)]\,10^6$$

$$= \frac{0.32}{207 \times 10^3}\,(6 + 2.9 + 0.575)$$

$$= \mathbf{14.7\ \mu m}$$

**Solution (b):** graphical

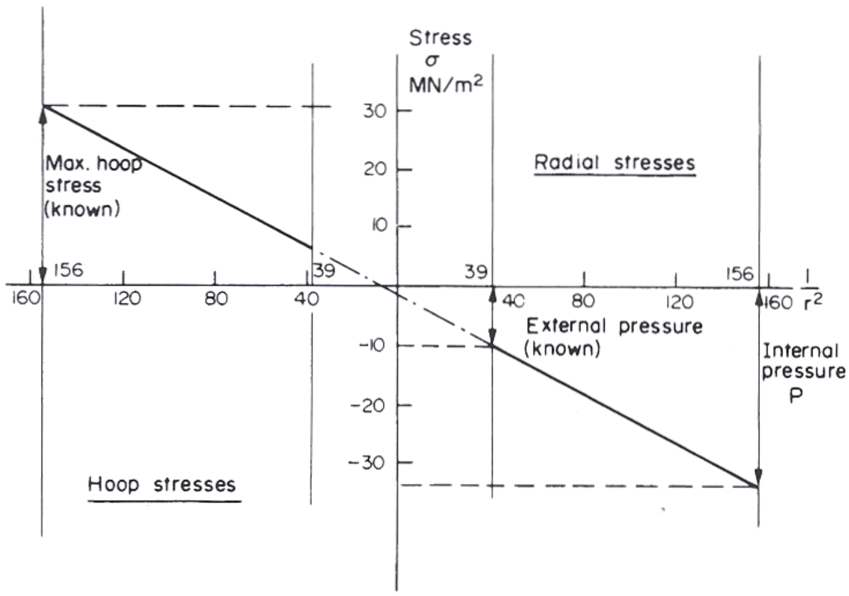


Fig.11

The graphical solution is shown in Fig. 11. The boundaries of the cylinder are as follows:

for $r = 0.08$ m, $\quad \frac{1}{r^2} = 156$

and for $r = 0.16$ m, $\quad \frac{1}{r^2} = 39$

The two fixed points on the graph which enable the line to be drawn are, therefore,

$$\sigma_r = -10 \text{ MN/m}^2 \text{ at } r = 0.16 \quad \text{and} \quad \sigma_H = 30 \text{ MN/m}^2 \text{ at } r = 0.08 \text{ m}$$

The allowable internal pressure is then given by the value of $\sigma_r$ at $r = 0.08$ m $\left(\frac{1}{r^2} = 156\right)$, i.e. 34 MN/m$^2$.

Similarly, the hoop stress at the outside surface is given by the value of $\sigma_H$ at $\frac{1}{r^2} = 39$, i.e. 6 MN/m$^2$, and the longitudinal stress by the intercept on the $\sigma$ axis, i.e. 2 MN/m$^2$ compressive.

N.B. – In practice all these values should be calculated by proportions.

**3.** (a) In an experiment on a thick cylinder of 100 mm external diameter and 50 mm internal diameter the hoop and longitudinal strains as measured by strain gauges applied to the outer surface of the cylinder were 240 x $10^{-6}$ and 60 x $10^{-6}$, respectively, for an internal pressure of 90 MN/m$^2$, the external pressure being zero.
Determine the actual hoop and longitudinal stresses present in the cylinder if
E = 208 GN/m$^2$ and $\nu$ = 0.29. Compare the hoop stress value so obtained with thetheoretical value given by the Lame equations.
(b) Assuming that the above strain readings were obtained for a thick cylinder of 100 mm external diameter but unknown internal diameter calculate this internal diameter.

**Solution**

a) $$\varepsilon_H = \frac{1}{E}(\sigma_H - \nu\sigma_L) \quad \text{and} \quad \varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_H)$$

since $\sigma_r = 0$ at the outside surface of the cylinder for zero external pressure.

$$\therefore \quad 240 \times 10^{-6} \times 208 \times 10^{9} = \sigma_H - 0.29\sigma_L = 50 \times 10^{6} \qquad (1)$$

$$60 \times 10^{-6} \times 208 \times 10^{9} = \sigma_L - 0.29\sigma_H = 12.5 \times 10^{6} \qquad (2)$$

(1) × 0.29 $\quad 0.29\sigma_H - 0.084\sigma_L = 14.5 \times 10^{6} \qquad (3)$

(2) $\quad \sigma_L - 0.29\sigma_H = 12.5 \times 10^{6}$

(3) + (2) $\quad 0.916\sigma_L = 27 \times 10^{6}$

$\therefore \quad \boldsymbol{\sigma_L = 29.5\ \mathrm{MN/m^2}}$

Substituting in (2) $\quad 0.29\sigma_H = 29.5 - 12.5 = 17 \times 10^{6}$

$\boldsymbol{\sigma_H = 58.7\ \mathrm{MN/m^2}}$

The theoretical values of $\sigma_H$ for an internal pressure of 90 MN/$m^2$ may be obtained from Fig. 12, the boundaries of the cylinder being given by r = 0.05 and r = 0.025,

i.e. $\quad \dfrac{1}{r^2} = 400$ and 1600 respectively

i.e. $\quad \boldsymbol{\sigma_H = 60\ \mathrm{MN/m^2}}$ theoretically

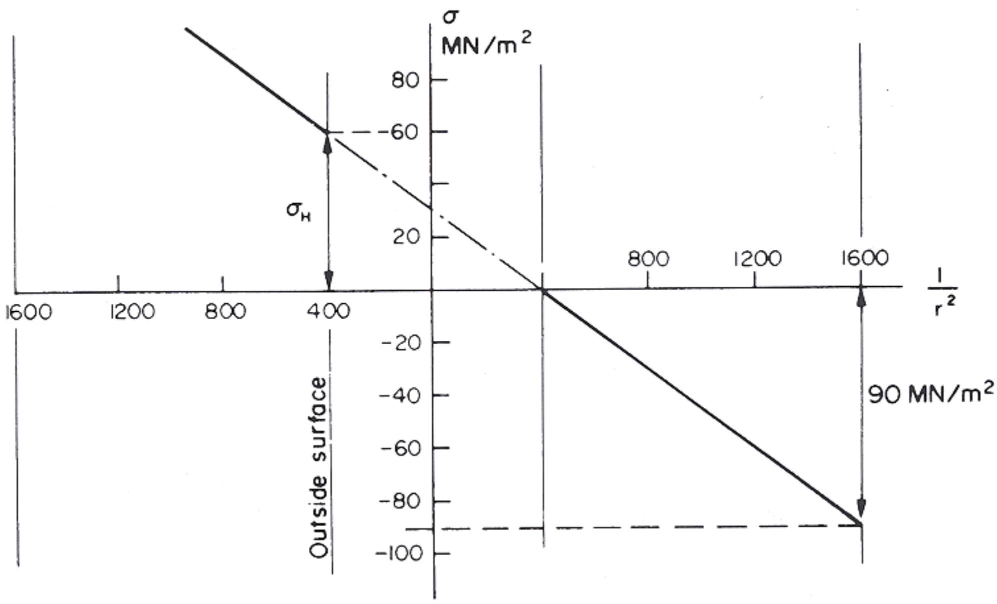


Fig.12

(b) From part (a) $\sigma_H = 58.7\ \text{MN/m}^2$ at $r = 0.05$

and $\sigma_r = 0$ at $r = 0.05$

$$\therefore \quad 58.7 = A + 400B$$

$$0 = A - 400B$$

Adding: $58.7 = 2A \quad \therefore \ A = 29.35$

and since $A = 400B \quad \therefore \ B = 0.0734$

Therefore for the internal radius $R_1$ where $\sigma_r = 90\ \text{MN/m}^2$

$$-90 = 29.35 - \frac{0.0734}{R_1^2}$$

$$R_1^2 = \frac{0.0734}{119.35} = 0.000615$$

$$= 6.15 \times 10^{-4}$$

$$\therefore \quad R_1 = 2.48 \times 10^{-2}\ \text{m} = 24.8\ \text{mm}$$

Internal diameter = 49.6 mm

For a graphical solution of part (b), see Fig.13, where the known points which enable the Lame line to be drawn are, as above:

$$\sigma_H = 58.7 \quad \text{at} \ \frac{1}{r^2} = 400 \quad \text{and} \quad \sigma_r = 0 \quad \text{at} \ \frac{1}{r^2} = 400$$

It is thus possible to determine the value of $1/R_1^2$ which will produce $\sigma_r = -90\ \text{MN/m}^2$.

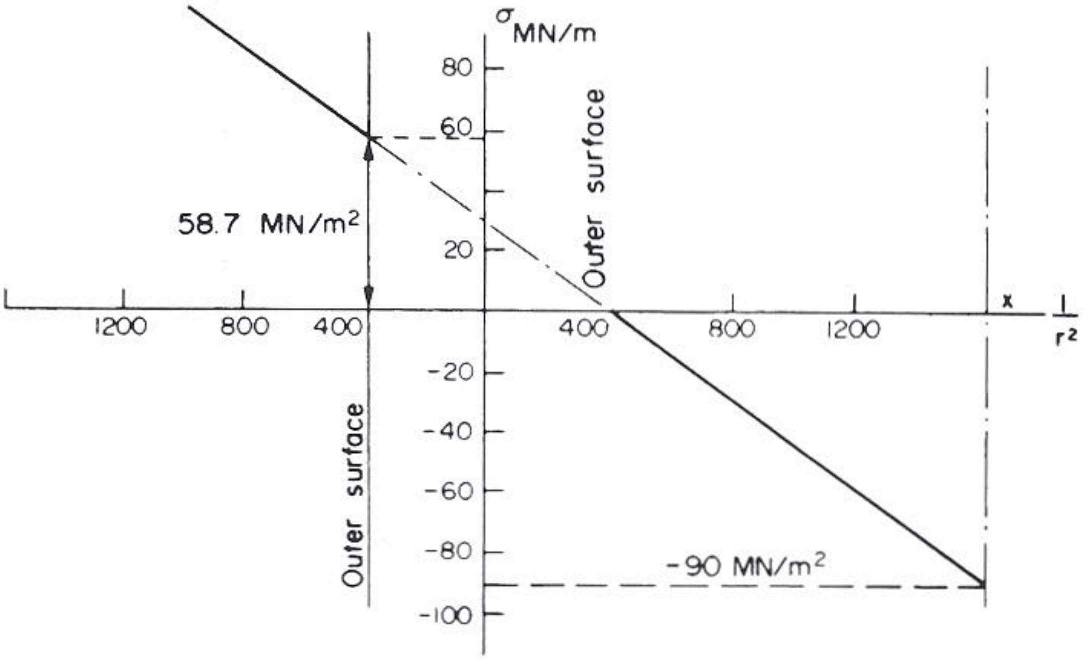


Fig.13

Let the required value of $\frac{1}{R_1^2} = x$

then by proportions $\frac{90}{x-400} = \frac{58.7}{800}$

$$x - 400 = \frac{90 \times 800}{58.7} = 1225$$

$$\therefore \quad x = 1625$$

$$\therefore \quad R_1 = 24.8 \text{ mm}$$

i.e. required internal diameter = **49.6 mm**

For a graphical solution of part (b), see Fig. 14, where the known points which enable the Lame line to be drawn are, as above:

$$\sigma_H = 58.7 \quad \text{at} \quad \frac{1}{r^2} = 400 \quad \text{and} \quad \sigma_r = 0 \quad \text{at} \quad \frac{1}{r^2} = 400$$

It is thus possible to determine the value of $1/R_1^2$ which will produce $\sigma_r$ = - 90 MN/m$^2$.

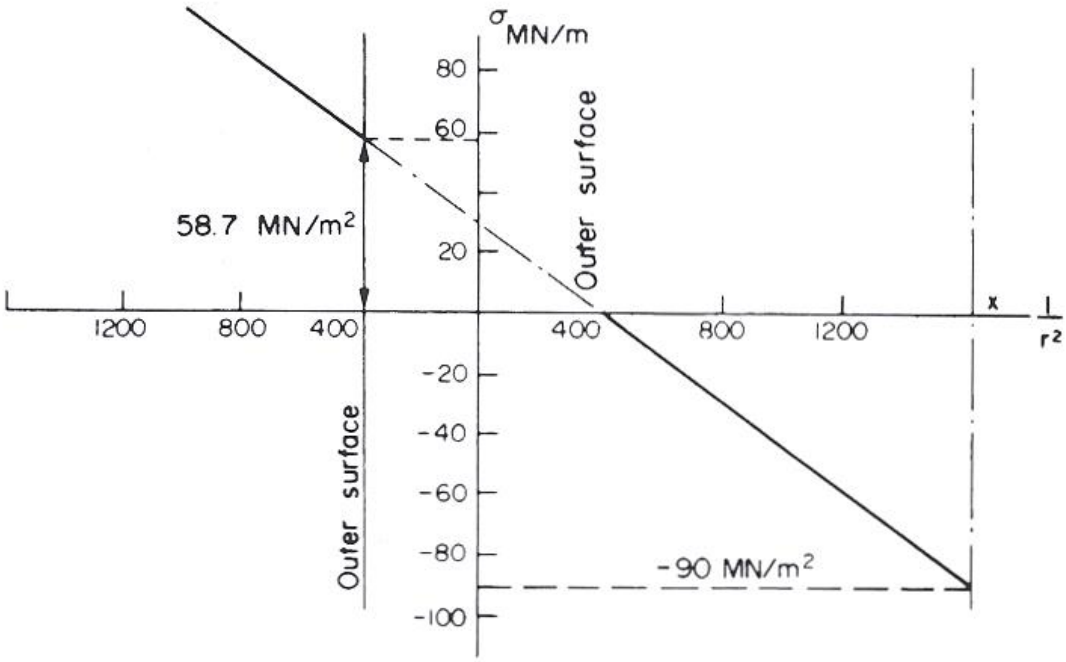


Fig. 14

Let the required value of $\dfrac{1}{R_1^2} = x$

then by proportions $\quad \dfrac{90}{x-400} = \dfrac{58.7}{800}$

$$x - 400 = \frac{90 \times 800}{58.7} = 1225$$

$\therefore \quad x = 1625$

$\therefore \quad R_1 = 24.8 \text{ mm}$

i.e. required internal diameter = **49.6 mm**

**4.** A compound cylinder is formed by shrinking a tube of 250 mm internal diameter and 25 mm wall thickness onto another tube of 250 mm external diameter and 25 mm wall thickness, both tubes being made of the same material. The stress set up at the junction owing to shrinkage is 10 $MN/m^2$. The compound tube is then subjected to an internal pressure of 80 $MN/m^2$. Compare the hoop stress distribution now obtained with that of a single cylinder of 300 mm external diameter and 50 mm thickness subjected to the same internal pressure.

**Solution (a): analytical**

A solution is obtained as described in § 9.7, i.e. by considering the effects of shrinkage and internal pressure separately and combining the results algebraically.

**Shrinkage only- outer tube**

At $r = 0.15$, $\sigma_r = 0$ and at $r = 0.125$, $\sigma_r = -10\ \mathrm{MN/m^2}$

$$\therefore \qquad 0 = A - \frac{B}{0.15^2} = A - 44.5B \qquad (1)$$

$$-10 = A - \frac{B}{0.125^2} = A - 64B \qquad (2)$$

Subtracting (1) - (2), 10 = 19.5B $\therefore$ B = 0.514

Substituting in (1), A = 44.5B $\therefore$ A = 22.85

$\therefore$ hoop stress at 0.15 m radius = A + 44.5B = 45.7 $MN/m^2$

hoop stress at 0.125 m radius = A + 64B = 55.75 $MN/m^2$

**Shrinkage only - inner tube**

At r = 0.10, $\sigma_r = 0$ and at r = 0.125, $\sigma_r = -10$ $MN/m^2$

$$\therefore \qquad 0 = A - \frac{B}{0.1^2} = A - 100B \qquad (3)$$

$$-10 = A - \frac{B}{0.125^2} = A - 64B \qquad (4)$$

Subtracting (3) - (4), 10 = - 36B $\therefore$ B = - 0.278

Substituting in (3), A = 100B $\therefore$ A = - 27.8

$\therefore$ hoop stress at 0.125 m radius = A + 64B = - 45.6 MN/m$^2$

hoop stress at 0.10 m radius = A + 100B = - 55.6 MN/m$^2$

Considering internal pressure only (on complete cylinder)

At $r = 0.15$, $\sigma_r = 0$ and at $r = 0.10$, $\sigma_r = -80$

$$\therefore \quad 0 = A - 44.5B \tag{5}$$

$$-80 = A - 100B \tag{6}$$

Subtracting (5) − (6), $80 = 55.5B$ $\therefore$ $B = 1.44$

From (5), $A = 44.5B$ $\therefore$ $A = 64.2$

$\therefore$ At $r = 0.15$ m, $\sigma_H = A + 44.5B = 128.4$ MN/m$^2$

$r = 0.125$ m, $\sigma_H = A + 64B = 156.4$ MN/m$^2$

$r = 0.1$ m, $\sigma_H = A + 100B = 208.2$ MN/m$^2$

The resultant stresses for combined shrinkage and internal pressure are then:

outer tube: $r = 0.15$ $\sigma_H = 128.4 + 45.7 = \mathbf{174.1}$ **MN/m$^2$**

$r = 0.125$ $\sigma_H = 156.4 + 55.75 = \mathbf{212.15}$ **MN/m$^2$**

inner tube: $r = 0.125$ $\sigma_H = 156.4 - 45.6 = \mathbf{110.8}$ **MN/m$^2$**

$r = 0.1$ $\sigma_H = 208.2 - 55.6 = \mathbf{152.6}$ **MN/m$^2$**

**Solution (b): graphical**

The graphical solution is obtained in the same way by considering the separate effects of shrinkage and internal pressure as shown in Fig.15.

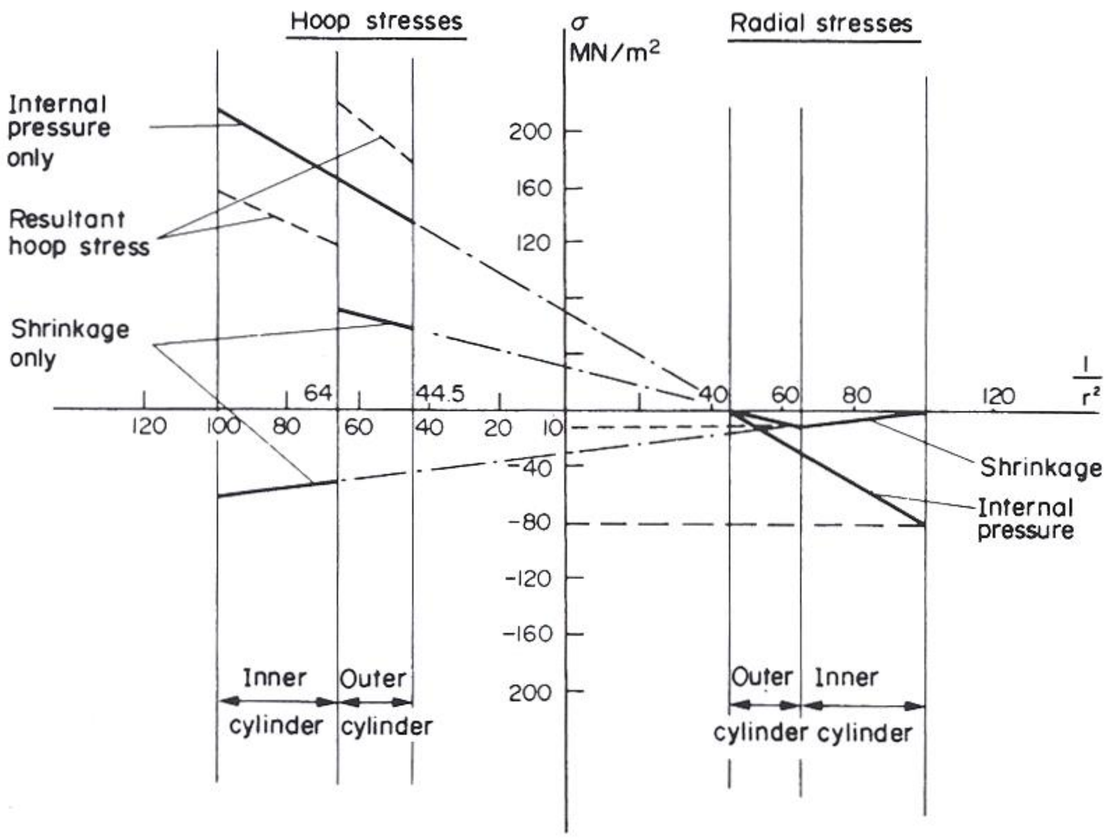


Fig. 15

The final results are illustrated in Fig. 16 (values from the graph again being determined by proportion of the figure).

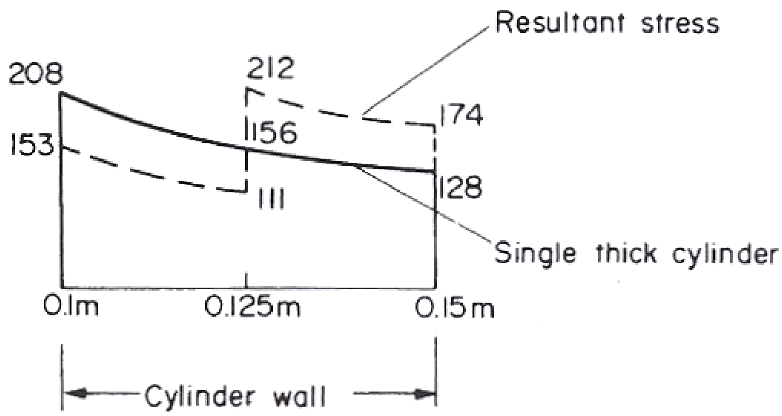


Fig. 16